Geometry 11.1 Angle Measures in Polygons (pp 661-664)

Draw examples of 3-sided, 4-sided, 5-sided, and 6-sided convex polygons. In each polygon, draw all the diagonals from one vertex. Notice that this divides each polygon into triangular regions. Complete the table below. What is the pattern in the sum of the measures of the interior angles of any convex n-gon?

Polygon	Number of Sides	Number of triangles	Sum of the measures of the interior Angles
Triangle	3	1	180
Quadrilateral			
Pentagon			
Hexagon			
Decagon			
n-gon			

THEOREMS ABOUT INTERIOR ANGLES

THEOREM 11.1 ***Polygon Interior Angles Theorem***

The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \cdot 180°$.

COROLLARY TO THEOREM 11.1

The measure of each interior angle of a regular n-gon is

$$\frac{1}{n} \cdot (n - 2) \cdot 180°, \text{ or } \frac{(n - 2) \cdot 180°}{n}.$$

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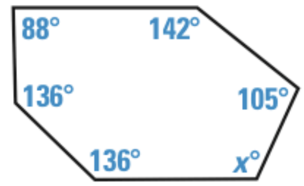
EXAMPLE 1 ***Finding Measures of Interior Angles of Polygons***

Find the value of x in the diagram shown.



SOLUTION

The sum of the measures of the interior angles of any hexagon is $(6 - 2) \cdot 180° = 4 \cdot 180° = 720°$.

Add the measures of the interior angles of the hexagon.

$$136° + 136° + 88° + 142° + 105° + x° = 720° \quad \text{The sum is 720°.}$$

$$607 + x = 720 \quad \text{Simplify.}$$

$$x = 113 \quad \text{Subtract 607 from each side.}$$

▶ The measure of the sixth interior angle of the hexagon is 113°.

2. Guided Practice: A heptagon has four interior angles that measure 160° each and two interior angles that are right angles. What is the measure of the other interior angle?

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3. Example: The measure of each interior angle of a regular polygon is 165°. How many sides does the polygon have?

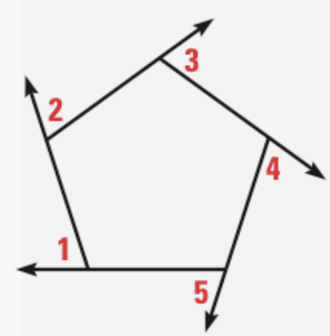
Statement	Reason
$\frac{180(n-2)}{n} = \text{Interior Angle of a regular polygon}$	
$\frac{180(n-2)}{n} = 165$	Given
$180(n-2) = 165n$	Multiplication property of equality
180n - 360 = 165n	Distributive property
-360=-15n	Subtraction property of equality
24=n	Division property of equality
A a regular polygon with each interior angle measuring $165°$ will have 24 sides.	

4. Guided Practice: Find the measure of each interior angle in a regular 11-gon.

THEOREMS ABOUT EXTERIOR ANGLES

THEOREM 11.2 ***Polygon Exterior Angles Theorem***

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.



COROLLARY TO THEOREM 11.2

The measure of each exterior angle of a regular n-gon is $\frac{1}{n} \cdot 360°$, or $\frac{360°}{n}$.

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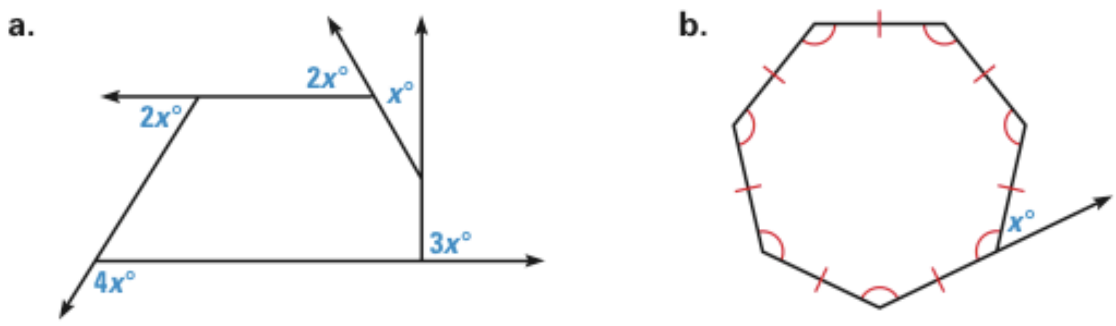
EXAMPLE 3 *Finding the Measure of an Exterior Angle*

Find the value of x in each diagram.



SOLUTION

a. $2x° + x° + 3x° + 4x° + 2x° = 360°$ — Use Theorem 11.2.

$12x = 360$ — Combine like terms.

$x = 30$ — Divide each side by 12.

b. $x° = \frac{1}{7} \cdot 360°$ — Use $n = 7$ in the Corollary to Theorem 11.2.

≈ 51.4 — Use a calculator.

▶ The measure of each exterior angle of a regular heptagon is about 51.4°.

7. Guided Practice. The measure of each exterior angle of a regular polygon is 40°. How many sides does the polygon have?

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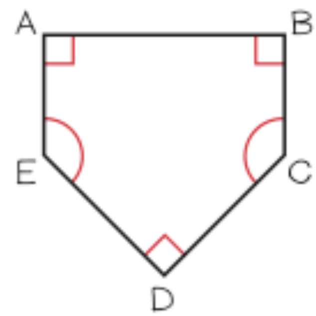
EXAMPLE 4 *Finding Angle Measures of a Polygon*

SOFTBALL A home plate marker for a softball field is a pentagon. Three of the interior angles of the pentagon are right angles. The remaining two interior angles are congruent. What is the measure of each angle?

SOLUTION

DRAW A SKETCH Sketch and label a diagram for the home plate marker. It is a nonregular pentagon. The right angles are $\angle A$, $\angle B$, and $\angle D$. The remaining angles are congruent. So $\angle C \cong \angle E$. The sum of the measures of the interior angles of the pentagon is 540°.



VERBAL MODEL

$$\boxed{\text{Sum of measures of interior angles}} = 3 \cdot \boxed{\text{Measure of each right angle}} + 2 \cdot \boxed{\text{Measure of } \angle C \text{ and } \angle E}$$

LABELS

Sum of measures of interior angles = **540** (degrees)

Measure of each right angle = **90** (degrees)

Measure of $\angle C$ and $\angle E$ = $\boldsymbol{x}$ (degrees)

REASONING

$\mathbf{540} = 3 \cdot \mathbf{90} + 2\boldsymbol{x}$	Write the equation.
$540 = 270 + 2x$	Simplify.
$270 = 2x$	Subtract 270 from each side.
$135 = x$	Divide each side by 2.

▶ So, the measure of each of the two congruent angles is 135°.

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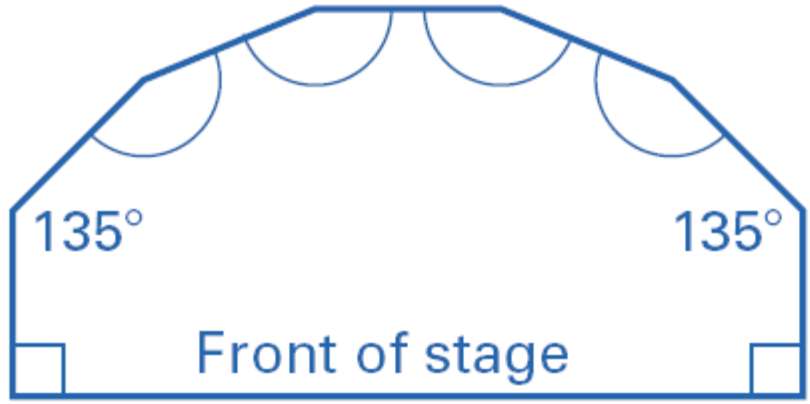
9. Guided Practice. Shown is the floor plan of an outdoor theater stage. The stage has two right angles and two angles that measure 135° each. The remaining interior angles are congruent. What is the measure of each of these angles?



EXAMPLE 5 ***Using Angle Measures of a Regular Polygon***

SPORTS EQUIPMENT If you were designing the home plate marker for some new type of ball game, would it be possible to make a home plate marker that is a regular polygon with each interior angle having a measure of (**a**) 135°? (**b**) 145°?

SOLUTION

a. Solve the equation $\frac{1}{n} \cdot (n - 2) \cdot 180° = 135°$ for n. You get $n = 8$.

▶ Yes, it would be possible. A polygon can have 8 sides.

b. Solve the equation $\frac{1}{n} \cdot (n - 2) \cdot 180° = 145°$ for n. You get $n \approx 10.3$.

▶ No, it would not be possible. A polygon cannot have 10.3 sides.

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<u>Guided Practice</u>.

10. Can a stage be built that is a regular polygon with each angle having a measure of 175°?

11. What is the exterior angle of a polygon?

12. Explain in words how find the measure of each interior angle and each exterior angle in regular polygon.

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